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Aerodynamics Technical Memorandum 379

A MATHEMATICAL MODEL OF THE SEA KING MK.50
HELICOPTER AERODYNAMICS AND KINEMATICS

by

M.J. Williams and A.M. Arney

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A MATHEMATICAL MODEL OF THE SEA KING MK.50 HELICOPTER
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SUMMARY

Details are given of the expressions used to describe the aerodynamics and kinematics of the Sea King Mk.50 helicopter during steady flight and low rate manoeuvres up to an advance ratio of 0.3. The aerodynamics/kinematics formulation is a major component of the Sea King mathematical model developed by Aeronautical Research Laboratories (ARL) for flight simulation of this Anti-Submarine Warfare helicopter.



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NOTATION

- A, B, C
 - Moments of inertia of helicopter about its respective body axes.
- A'_{1S}, B'_{1S}
 - Effective cyclic pitch control inputs with respect to shaft axes after allowing for control phasing and blade lag.
- A'_{1W}, B'_{1W}
 - Effective cyclic pitch control inputs with respect to wind axes.
- A_{1W}, B_{1W}
 - Total cyclic pitch variation observed in wind axes including pitch flap coupling, i.e. orientation of ANF axes with respect to wind axes.
- B_L, B_{LT}
 - Blade tip loss factors for main and tail rotors.
- C_{DTV}
 - Profile-drag coefficient of the vertical tail at an angle of attack of 90 degrees. (See §4, equation 3).
- $C_{DXF}, C_{DYF}, C_{DZF}$
 - Profile-drag coefficients of the fuselage when moving in the x_H , y_H and z_H directions, respectively. (See §4, equations 1,2 and 5 respectively).
- $C_H, C_{H_H}, C_{H_S}, C_{H_W}$
 - Coefficients of main rotor force components along the negative senses of the Ox_{ANF} , Ox_H , Ox_S and Ox_W axes, respectively.
(C_H = Main Rotor Drag Force/ q_S).

NOTATION (CONT.)

- C_L, C_{L_H}, C_{L_W} Coefficients of main rotor moment components (positive for right-handed sense) about the Ox_{ANF} , Ox_H and Ox_W axes, respectively.
($C_L = L/q_S R$).
- C_M, C_{M_H}, C_{M_W} Coefficients of main rotor moment components (positive for right-handed sense) about the Oy_{ANF} , Oy_H and Oy_W axes respectively.
($C_M = M/q_S R$)
- C_N normal force coefficient of horizontal tail plane. (See § 4, equation 6).
- $C_Q, C_{Q_H}, C_{Q_S}, C_{Q_W}$ Coefficients of main rotor moment components (positive for right-handed sense) about the Oz_{ANF} , Oz_H , Oz_S and Oz_W axes, respectively.
($C_Q = Q/q_S R$)
- C_{Q_T} tail rotor torque coefficient.
($C_{Q_T} = Q_T/q_{ST} R_T$)
- $C_T, C_{T_H}, C_{T_S}, C_{T_W}$ Coefficients of main rotor force components along the negative senses of the Oz_{ANF} , Oz_H , Oz_S and Oz_W axes, respectively.
($C_T = \text{Main Rotor Thrust}/q_S$)
- C_{T_T} tail rotor thrust coefficient.
($C_{T_T} = \text{Tail Rotor Thrust}/q_{ST}$)
- $C_Y, C_{Y_H}, C_{Y_S}, C_{Y_W}$ Coefficients of main rotor force components along the positive senses of the Oy_{ANF} , Oy_H , Oy_S and Oy_W axes, respectively.
($C_Y = \text{Main Rotor Side Force}/q_S$)
- EK_1 Factor relating downwash at fuselage to induced flow through the rotor disk.

NOTATION (CONT.)

$F_{G\text{LOSS}}$	- Factor representing losses in transmission system and accessory drives.
G_{RAT}	- Ratio of tail rotor rotational speed to that of main rotor.
H_{ROT}	- Height of rotor above ground.
$(H_{\text{ROT}})_0$	- Height of rotor when aircraft is on ground.
I_T	- Total effective inertia of rotors and transmission about main rotor shaft.
K_{TV}	- Constant used in calculations of vertical tail force.
L, M, N	- Total aerodynamic moments about respective Ox_H, Oy_H, Oz_H body axes.
$L_F, L_{\text{TV}}, L_{\text{TH}}, L_T$	- Rolling moment contributions arising from fuselage, vertical tail fin, horizontal tail and tail rotor.
M_F	- Pitching moment contribution arising from fuselage.
M_{TH}	- Pitching moment contribution arising from horizontal tail.
M_{TIP}	- Blade tip Mach number.
N_F	- Yawing moment contribution arising from fuselage and tailplane.
N_r	- Yawing moment contribution due to yaw rate - $(\partial N / \partial r)_{\text{HEH}}$

NOTATION (CONT.)

N_T - Yawing moment contribution arising from tail rotor.

$OLD A_{1S}$ - Control system cyclic and collective pitch outputs.

$OLD B_{1S}$

$OLD \theta_c$

Q - Aerodynamic torque acting on main rotor about its $O_{Z_{ANF}}$ axis; positive retarding.

Q_{CMP} - Compressibility torque, main rotor.

Q_{ENG} - Torque supplied by engine.

Q_{LOAD} - Total load on engine arising from main rotor, tail rotor, accessories and transmission losses.

Q_{MAX} - Engine torque at maximum power and specified speed.

Q_x - Engine torque as registered by aircraft instrument.

Q_T - Aerodynamic torque acting on tail rotor shaft.

$(Q_T)_{CMP}$ - Compressibility torque, tail rotor.

R, R_T - Tip radius of main and tail rotor respectively.

$(RPM)_{SET}$ - Specified rotor r.p.m. at which maximum engine power is delivered.

NOTATION (CONT.)

$(SHP)_{\max}$	- Maximum engine power.
S_{TH}, S_{TV}	- Areas of horizontal and vertical tail surfaces.
$(SC_D)_{\alpha_M}$	- Coefficient of quadratic terms in expressions for fuselage longitudinal force. (See §4, equation 1).
$(SC_D)_{u/c}, (SC_D)_{wep}$	- Equivalent flat plate drag contribution arising from undercarriage and externally carried weapons (Torpedoes). (See §4, equation 1).
$(SC_L)_{\alpha_z}$	- Coefficient of angle of attack term in expression for fuselage normal force: $= S \frac{dC_L}{d\alpha_z}$ (See §4, equation 5).
$(SC_M)_{BOD}$	Equivalent pitching moment coefficient
$(SC_M)_{u/c}$	- contributions arising from fuselage, undercarriage and weapons. (See §4, equation 10).
$(SC_M)_{wep}$	
S_x, S_y, S_z	- Projected areas of fuselage in body axes directions.
T_{XH}, T_{YH}, T_{ZH}	- Components of dunking sonar cable tension in body axes.
V	- Helicopter airspeed.
\bar{V}	- non-dimensionalized helicopter airspeed.
V_T	- Resultant airspeed at tail rotor hub.
\bar{V}_T	- Non-dimensionalized airspeed at tail rotor hub.

NOTATION (CONT.)

V_{WE}	- Horizontal wind speed relative to Earth.
X, Y, Z	- Resultant forces along body axes.
X_F, Y_F, Z_F	- Components of aerodynamic force on the fuselage due to motion of the helicopter along the respective body axes.
Y_T	- Effective side force arising from tail rotor thrust after allowance for fin blockage.
Y_{TV}	- Side force contribution due to vertical tail fin.
Z_{TH}	- Normal force contribution of horizontal tail plane.
a, a_T	- Lift curve slopes of blade sections of main and tail rotor respectively.
a_0, a_{0T}	- Coning angles for main and tail rotor.
a_{XH}, a_{YH}, a_{ZH}	- Inertial acceleration components in respective body axes.
a_s	- Speed of sound.
a_{TV}	- Lift curve slope of vertical tail fin.
a_1, b_1	- Longitudinal and lateral cyclic flapping coefficients of main rotor.
b, b_T	- Number of blades in main and tail rotor.
d_{TH}	- Distance of horizontal tail plane centre of pressure from roll axis.

NOTATION (CONT.)

- f_4 - empirically based constant used in expressions for fuselage normal force.
- $f(z)$ - Ground effect factor used in main rotor aerodynamics.
- $f(\mu_x)$ - Function of fuselage advance ratio, used in estimation of rotor downwash on horizontal tail plane.
- g - acceleration due to gravity.
- h_R, h_T - Respective heights of main and tail rotor hubs above c.g.
- h_F - Height above c.g. of point of application of fuselage side force; used in rolling moment estimation.
- h_{TV} - Height of vertical tail centre of pressure above roll axis.
- i - Incidence setting of horizontal tail plane; positive for leading edge up.
- k_{a_1} - Factor applied to longitudinal flapping angle (a_1).
- k_i - Induced velocity parameter associated with main rotor.
- $(k_i)_T$ - Induced velocity parameter associated with tail rotor.
- k_3, k_5 - Coefficients representing pitch change due to blade flapping and lagging respectively.

NOTATION (CONT.)

k_4	- Constant relating steady state blade lag with rotor aerodynamic torque.
l_T	- Distance of tail rotor from yawing axis.
l_{TH}	- Distance of horizontal tail plane centre of pressure from pitch axis.
l_{TV}	- Distance of vertical tail centre of pressure from yaw axis.
m	- All-up mass of helicopter.
p, q, r	- Angular velocity components.
q_S, q_{ST}	- Dimensionalizing factors applied to force coefficients of main and tail rotors, $q_S = \rho \pi R^2 (\Omega R)^2 \text{ and}$ $q_{ST} = \rho \pi R_T^2 (\Omega_T R_T)^2$
u, v, w	- Linear velocity components.
w_{eff}	- Effective normal velocity at horizontal tail plane.
w_1	- Normal velocity component induced by rotor downwash.
$(x, y, z)_{HEE}$	- Position co-ordinates of helicopter with respect to earth referenced axis.
x_{CH}	- Co-ordinate of c.g. location forward of fuselage datum.

NOTATION (CONT.)

- x_{yfm} - Distance behind c.g. of point of application of fuselage side force; used in yawing moment estimation.
- $(\phi, \theta, \psi)_{HE}$ - Matrices defined in Appendix 2 representing axis transformation through angles ϕ_{HE} , θ_{HE} , ψ_{HE} respectively.
- Ω, Ω_T - Shaft angular velocity of main and tail rotors.
- α - Fuselage angle of attack.
- α_w, α_T - Angle of attack between apparent wind vector and no-feathering planes of main and tail rotors respectively.
- α_D - Airframe incidence for minimum drag.
- α_L - Airframe incidence for zero lift.
- α_M - Incidence w.r. to minimum drag incidence,
 $= \alpha - \alpha_D$
- α_Z - Effective fuselage angle of attack.
- α_{TH}, α_{TV} - Flow angle of attack for horizontal and vertical tail surfaces.
- β - Fuselage angle of sideslip, also, blade flapping angle.
- γ, γ_T - Lock numbers for main and tail rotors.
- δ - Mean profile drag coefficient for main and tail rotor blade sections (both NACA 0012).

NOTATION (CONT.)

- δ_3, α_1 - Angles associated with respective flapping and lagging hinges which give coupled pitch responses to blade flapping and lagging motions.
- ξ_0, ξ_{0T} - Flap hinge radius as a fraction of blade tip radius for main and tail rotors.
- ξ_1, ξ_{1T} - Blade root radius as a fraction of blade tip radius for main and tail rotors.
- θ_c - Effective main rotor collective pitch at zero hub radius (control system input is OLD θ_c).
- θ_{SH} - Tilt angle of main rotor shaft relative to body axis, tilt normally negative.
- θ_T - Tail rotor collective pitch input from control system.
- θ_{TE} - Effective tail rotor collective pitch.
- θ_1 - Blade twist between hub and tip; negative for usual case where pitch decreases towards tip.
- κ - Control phasing angle.
- λ, λ_T - Non-dimensional total flow averaged across the no-feathering planes of main and tail rotor; positive in the positive sense of rotor thrust.
- λ_1 - Coefficient describing linear variation of λ in longitudinal direction.
- μ, μ_T - Advance ratio for main and tail rotors.

NOTATION (CONT.)

u_x	- Fuselage advance ratio, u_{HWH}/QR
v, v_T	- Mean induced velocity at N.F.P. of main and tail rotors; positive in sense opposite to positive sense of rotor thrust.
v_{HOV}, v_{HOVT}	- Mean induced velocity of main and tail rotors at hover, as given by momentum theory.
\bar{v}, \bar{v}_T	- Non-dimensional induced velocity at N.F.P. e.g. $\bar{v} = v/v_{HOV}$.
v_{LAG}	- Lagged value of v for calculation of transient inflow ratio.
ξ	- Steady state blade lag angle.
ρ	- Air density.
σ, σ_T	- Solidity of main and tail rotors.
τ	- Time constant for calculation of v_{LAG} .
τ_1, τ_2, τ_3	- Time constants in engine model.
$(\phi, \theta, \psi)_{HE}$	- Euler angles expressing aircraft attitude with respect to Earth axis.
ψ	- Blade azimuth angle measured in direction of rotation (zero at downwind position).
ψ_{WEE}	- Earth axis azimuth angle defining direction from which horizontal wind component comes.
ψ_{WH}	- Azimuthal rotation of W-axis from S-axis.

NOTATION (CONT.)

ψ_{WEFF} - Effective azimuthal angle between w-axis and axis defined by cyclic control inputs.

χ_{ANF} - Wake sweep-back angle in ANF axis system.

χ_{FA} , χ_{LAT} - Wake sweepback components in body axis fore-aft and lateral directions.

Abbreviations

ANF - axis of no feathering

NFP - no feathering plane

TPP - tip path plane

c.g. - centre of gravity

r.p.m. - revolutions per minute

Suffixes

Vector components not specifically given in NOTATION are identified by the use of triple subscripts.

(a) The first subscript indicates to which quantity the vector component belongs, using the notation

H - helicopter centre of gravity

T - tail rotor hub

W - ambient wind.

(b) The second indicates the datum to which the vector is related, where

E - Earth axis

W - ambient air.

NOTATION (CONT.)

(c) The third indicates in which axis system the vector is resolved, where

E = earth axis

H = helicopter body axis

S = helicopter/shaft axis

T = tail rotor axis

W = wind/shaft axis.

1. INTRODUCTION

A mathematical model of the Sea King helicopter in the ASW mode has been developed by ARL in response to a Royal Australian Navy (RAN) Task requirement. General descriptions of this model have been reported in Refs. 1 and 2. A more detailed description of one of its major components, namely the aerodynamics/kinematics model is the subject of this report. The other major components are the control systems and cable/sonar transducer models which have been described in References 3-6 and 7,8 respectively. The earliest version of the Sea King model resulted essentially from the substitution of relevant Sea King data into Packer's model for the Wessex helicopter described in Refs. 9-11. Because the Wessex model was initially intended for simulation of flight behaviour in near-hover conditions and at low advance ratios, $\mu = 0.1$ or so, certain simplifications had been made in regard to the induced flow and fuselage aerodynamics. For these and other reasons to be discussed later, the initial Sea King model has been modified by the introduction of new features described herein, which will meet the requirement for adequate simulation up to at least $\mu = 0.3$.

Generally, the essential structure of the original program of Ref. 9 has been retained together with its unique subscripting system for identifying vector and attitude components. In addition to the modifications previously mentioned, several other features have been added to give a better representation of the Sea King, such as main rotor pitch flap coupling and cyclic control mixing. The incorporation of a simple engine model was also found to be necessary in order to account for large excursions of rotor speed and torque observed in flight under certain conditions. An effort has also been made to model the high transient loads arising from 'inflow damping' when rapid collective pitch changes are introduced in flight.

This aerodynamics/kinematics model is a dynamic three-dimensional representation of body motion with six degrees of freedom. Main and tail rotor flapping motions are calculated but the rotor is considered quasi-static, in that the disk is assumed to respond instantaneously to the motion of the fuselage. The inclusion of cross-coupling terms, such as arise from angular rates, means that the model is not limited to small perturbation studies. Being derived from the Wessex

model of Ref. 9, the present model is also expected to be capable of predicting low rate (< 5 rad/s) dynamic response manoeuvres. The validity of the model is presently being assessed by comparing model predictions with results from A.R.L. flight trials of the Sea King Mk50 as reported in Reference [12].

Section 2 of this report gives a general description of the Sea King Math Model. The model structure allows convenient subdivision into self contained topics which are commented on in the following sections. Section 3 describes the main and tail rotor aerodynamics, while the fuselage aerodynamics are given in Section 4. The equations of motion of a rigid body with rotating sub systems are detailed in Section 5 and the kinematic equations are stated in Section 6. A description of the simple engine model used is given in Section 7, followed by a discussion of the control inputs in Section 8. Throughout this report, discussion is limited to the assumptions, approximations and physical aspects related to the formulation of expressions used in the coding. Detailed derivations of expressions which may be found in standard texts and references are not given.

2. GENERAL DESCRIPTION OF MODEL

The aerodynamics/kinematics model constitutes one of 3 major elements of the complete Sea King mathematical model, the others being the Systems and Cable models respectively. The computer program for the complete model is written in CSMP-10 (ARL) [13 & 14]. This is a block oriented simulation language in which the model is expressed in terms of linked modules or blocks which represent a particular function or operation. The language also incorporates "user-defined" blocks written as Fortran subroutines which allow complex algebraic expressions to be handled conveniently. The Aerodynamics, Systems and Cable models are such "user-defined" subroutines.

A schematic diagram of the Aerodynamics/Kinematics model is shown in Fig. 1. Motion of the aircraft is obtained by applying the equations of motion after all force and moments contributions at the aircraft c.g. have been determined. These components arise from the main rotor, tail rotor and fuselage/empennage aerodynamics which may include rotor downwash interference effects. Axis transformations are required so

that all contributions are converted to a common axis system before summation. For instance, main and tail rotor quantities are initially derived in the usual axes of no-feathering (ANF) centred at the respective rotor hubs. Details of the various axis systems used in this program may be found in Appendix 1.

Aircraft accelerations obtained from the equations of motion are integrated* to give both linear and angular velocity components in the body axes system. The orientation of this system with respect to Earth based axes is expressed in terms of Euler angles. These angles are obtained from integration of the Euler rates given by the solution of the kinematic equations. Transformations employing Euler angles are used to:-

- (a) derive aircraft velocity components in Earth axes whence by integration the position co-ordinates are obtained;
- (b) express Earth-referenced ambient wind conditions in body-referenced wind-velocity components. Knowing the aircraft velocity, relative airspeed components may be deduced.

Calculation of the required aerodynamic quantities associated with the main and tail rotors requires a knowledge of the inflow and advance ratios for each rotor. These are derived from the local airspeed vector and its angle of attack with respect to the ANF for each rotor. Thus, the previously determined body airspeed components must be transformed into components appropriate to each ANF axis system via the wind shaft axes. The angular orientation of the ANF to these axes is determined by cyclic pitch control inputs together with pitch contributions arising from pitch-flap coupling as explained in Section 8. In the case of the tail rotor no cyclic control inputs can be applied so that all cyclic pitch changes stem from flapping. Cyclic control inputs to the main rotor must also be referred from body to wind shaft axis after due allowance for control phasing. (See Section 8).

* Integrator blocks in CSMP-10 (ARL) normally use a second-order Runge-Kutta method. An Adams predictor-corrector integration method is also available [14].

This latter feature is introduced in the swash plate/rotor head geometry in order to give a more nearly longitudinal tip path plane tilt in response to longitudinal stick displacements.

The mean induced flow component of the total inflow through the rotor is based on momentum theory together with the Glauert hypothesis for forward flight (Ref. 9, page 21). It is also assumed that there is a linear variation of induced flow in the streamwise direction, the magnitude of which depends on the wake sweepback angle.

Equations for the forces and moments developed by flow over the fuselage and tail surfaces are generally based on those used in the Sea King Flight Simulator [15] but rewritten in non-dimensional form. A downwash-modifying function dependent on wake angle, similar to that employed in Ref. 15, is used to adjust pitching moments as a function of speed, so that agreement with flight values of pitch attitude is improved.

By its nature, the simplified engine model does not require incorporation in the aerodynamics/kinematics "user defined subroutine" and is written directly in the simulation language CSMP-10(ARL).

3. ROTOR AERODYNAMICS

3.1 Main Rotor Aerodynamics

Using blade element theory, expressions for hub forces and moments in ANF axes may be obtained by integration of the blade element forces with respect to radius and azimuth, as given in Ref. 9 and standard texts. The assumptions of Ref. 9 have been retained and are -

- (a) Blades are infinitely stiff in torsion and bending and hence second order flapping can be justifiably ignored.
- (b) The effects of reverse flow, blade stall and compressibility effects are not accounted for in the blade element analysis. At the highest aircraft speed of 120 kn ($\mu = 0.3$) these effects are likely to become evident. (An empirically based compressibility correction has been included in the expressions for rotor torque given in §7).

(c) Small angle approximations are used in respect of inflow, blade incidence and flapping motion angles.

Final expressions for the rotor forces and moments, which are given later in this section, employ non-dimensional flow velocities, λ and μ , normal and tangential to the no-feathering plane (NFP) respectively. Relations used to derive these quantities in the model program are now given -

1. Total airspeed vector,

$$V = \sqrt{(u_{HWW}^2 + w_{HWW}^2)}$$

2. Angle of incidence of flow to NFP,

$$\alpha_w = \arctan \left[\frac{w_{HWW}}{u_{HWW}} \right] = B_{1W}$$

3. Advance ratio,

$$\mu = V \cos \alpha_w / \Omega R$$

4. Inflow ratio (steady state),

$$\begin{aligned} \lambda &= (V \sin \alpha_w - v) / \Omega R \\ &= \mu \tan \alpha_w - v / \Omega R \end{aligned}$$

5. Transient inflow ratio (for improved modelling of rapid collective pitch changes),

$$\lambda = \mu \tan \alpha_w - v_{lag} / \Omega R$$

$$\text{where } \frac{v_{lag}}{v} = \frac{1}{1+ts} \quad \text{in Laplace form.}$$

6. Mean induced velocity v is now calculated; the first step is to calculate the momentum theory value at hover, v_{HOV} ,

$$v_{HOV} = \Omega R \sqrt{(C_T/2)}$$

Next, the non-dimensional airspeed vector,

$$\bar{v} = v/v_{HOV}$$

Now, the implicit relation for non-dimensional induced velocity is solved,

$$\bar{v} = (\bar{v}^2 + \bar{v}^2 - 2\bar{v}\bar{v} \sin \alpha_w)^{-\frac{1}{2}}$$

from whence,

$$v = k_1 \bar{v} v_{HOV} / f(z)$$

where k_1 is an empirically based induced velocity parameter.

The ground effect factor $f(z)$ is calculated from the expression of Reference 16, namely

$$f(z) = \left\{ 1 - \left[\frac{R}{4H_{ROT}} \right]^2 \left[\frac{1}{1 + (V/v)^2} \right] \right\}^{-1}$$

where H_{ROT} , the rotor height above ground level, is given by,

$$H_{ROT} = -z_{HEE} + (H_{ROT})_0$$

where $-z_{HEE}$ represents the height of the aircraft above the ground and $(H_{ROT})_0$ is the height of the rotor when the aircraft is on the ground.

7. A longitudinal variation of induced flow (Glauert-type) is assumed using the parameter λ_1 as for Ref. 9 where,

$$\lambda_1 = x_{ANF} v/\Omega R$$

In the present model the proportionality factor is represented by the sweepback angle χ_{ANF} expressed in radians.

The wake sweepback angle is given by,

$$\chi_{ANF} = \arctan(\mu/-\lambda)$$

8. Blade tip loss factor

$$B_{LOSS} = 1 - \sqrt{(2C_T/b^2)}$$

9. Blade Flapping Motion

In steady flight, the blade-motion must be periodic and is therefore capable of being expressed in a Fourier series. Thus for rigid blades:

$$\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi$$

ignoring terms of second order and above.

Thus a_0 represents the blade coning angle,

a_1 represents the first order longitudinal flapping angle,

b_1 represents the first order lateral flapping angle.

The Main Rotor flapping coefficients are given by the following expressions

$$a_0 = \frac{\gamma}{8} \left(\theta_c(1 + \mu^2) + \theta_1 \left[\left(-\frac{4}{5} + \frac{2}{3} \mu^2 \right) \right] + \frac{4}{3} \lambda \right. \\ \left. + \frac{2}{3} \mu \frac{P_{HWW}}{\Omega} \right) - \frac{3}{2} (g - \dot{w}_{HEW}) / \Omega^2 R$$

(8)

$$a_1 = \frac{k_{a_1}}{1 - \mu^2/2} \{ \mu \left[-\frac{8}{3} \theta_c + 2 \theta_1 + 2 \lambda \right] + \frac{P_{HWW}}{\Omega} - \frac{16}{\gamma} - \frac{q_{HWW}}{\Omega} \}$$

with k_{a_1} being an empirically derived factor allowing for the discrepancy between flight data and theory.

$$b_1 = \frac{1}{1 + \mu^2/2} \{ \frac{4}{3} \mu a_0 + \lambda_1 - \frac{q_{HWW}}{\Omega} - \frac{16}{\lambda} \frac{P_{HWW}}{\Omega} \}$$

10. Thrust coefficient,

$$C_T = \frac{a_0}{2} \{ \theta_c \left[-\frac{1}{3} + \frac{\mu^2}{2} (1 - \zeta_1) \right] + \theta_1 \left[-\frac{1}{4} + \frac{\mu}{4} \right] + \frac{1}{2} \lambda + \frac{P_{HWW}}{\Omega} \frac{\mu}{4} \} B_L^3$$

where B_L^3 approximates the loss in rotor thrust arising from reduced lift in the blade tip region. Similar allowances are made to the induced drag components of the rotor in-plane forces given in the next section. Expressions given later for rotor hub moments are modified in a similar manner using the factor B_L^4 .

11. Rotor Force coefficients,

$$C_H = \frac{a_0}{2} \{ \frac{\delta \mu}{2a} + \left[\theta_c \left(\frac{a_1}{3} - \frac{\lambda}{2} \mu (1 - \zeta_1) \right) + \theta_1 \left(\frac{a_1}{4} - \frac{\mu \lambda}{4} \right) + \frac{a_0^2 \mu}{4} + \frac{3}{4} a_1 \lambda + \frac{a_1^2 \mu}{4} - \frac{a_0 b_1}{6} - \frac{q_{HWW}}{\Omega} \left(\frac{a_0}{6} + \frac{b_1}{16} \right) - \frac{P_{HWW}}{\Omega} \left(\frac{\theta_c}{6} + \frac{\theta_1}{8} + \frac{\lambda}{2} + \frac{\mu a_1}{16} \right) \right] B_L^3 \}$$

$$\begin{aligned}
C_Y &= \frac{a_0}{2} \left(\theta_C \left(\frac{b_1}{3} + \frac{\mu^2 b_1}{2} (1 - \zeta_1) - \frac{3}{4} a_0 \mu + \frac{q_{HWW}}{6\Omega} \right) \right. \\
&\quad + \theta_1 \left(\frac{b_1}{4} + \frac{b_1 \mu^2}{4} - \frac{a_0 \mu}{2} + \frac{q_{HWW}}{8\Omega} \right) \\
&\quad + a_0 \left(\frac{a_1}{6} - a_1 \mu^2 (1 - \zeta_1) - \frac{3}{2} \lambda \mu (1 - \zeta_1) \right) \\
&\quad + b_1 \left(\frac{3}{4} \lambda + \frac{\mu a_1}{4} \right) \\
&\quad \left. + \frac{q_{HWW}}{\Omega} \left(\frac{7}{16} a_1 \mu + \frac{\lambda}{2} \right) - \frac{P_{HWW}}{\Omega} \left(\frac{a_0}{6} - \frac{5}{16} b_1 \mu \right) \right\} B_L^3
\end{aligned}$$

12. Resolution of hub force coefficients (ANF axes) back to body axes for substitution in the equations of motion. Small angle approximations are used where appropriate.

(a) From ANF to wind axes via the cyclic blade angles, A_{1W} , B_{1W}

$$C_{TW} = C_T - C_Y A_{1W} + C_H B_{1W}$$

$$C_{HW} = C_H - C_T B_{1W}$$

$$C_{YW} = C_Y + C_T A_{1W}$$

(b) From wind to shaft axes via the azimuthal angle, ψ_{WH}

$$C_{TS} = C_{TW}$$

$$C_{HS} = C_{HW} \cos \psi_{WH} + C_{YW} \sin \psi_{WH}$$

$$C_{YS} = C_{YW} \cos \psi_{WH} - C_{HW} \sin \psi_{WH}$$

(c) From shaft to helicopter body axes via the shaft angle, θ_{SH}

$$C_{TH} = C_{TS} - C_{HS} \theta_{SH}$$

$$C_{HH} = C_{HS} + C_{TS} \theta_{SH}$$

$$C_{YH} = C_{YS}$$

(10)

13. Main Rotor torque coefficient in ANF axes -

$$\begin{aligned}
 C_Q = & \frac{a\sigma}{2} \left\{ \frac{\delta}{4a} (1 + \mu^2) + \left[- \left(\frac{\theta_c}{3} + \frac{\theta_1}{4} \right) \left(\lambda + \frac{\mu}{2} \frac{P_{HWW}}{\Omega} \right) \right. \right. \\
 & - \frac{\lambda^2}{2} - \frac{\mu^2}{4} \left(a_0^2 + \frac{3}{4} a_1^2 + \frac{b_1^2}{4} \right) - \frac{\mu}{2} \lambda a_1 \\
 & + \frac{\mu a_0}{3} \left(b_1 + \frac{q_{HWW}}{\Omega} \right) \\
 & - \frac{1}{8} \left(a_1^2 + b_1^2 + \frac{P_{HWW}^2}{\Omega^2} + \frac{q_{HWW}^2}{\Omega^2} - 2 a_1 \frac{P_{HWW}}{\Omega} \right. \\
 & \left. \left. + 2 b_1 \frac{q_{HWW}}{\Omega} \right] \right\} B_L^4
 \end{aligned}$$

14. Main Rotor hub moment coefficients in wind axes -

$$\begin{aligned}
 C_{L_W} &= \frac{3 a \sigma \zeta_0}{4 \gamma} \left\{ b_1 + A_{1W} - \gamma \left[- \frac{\mu}{3} \left(\theta_c + \frac{3}{4} \theta_1 + \lambda \right) - \frac{a_1}{9} \right] \right\} B_L^4 \\
 C_{M_W} &= \frac{3 a \sigma \zeta_0}{4 \gamma} \left\{ a_1 - B_{1W} + \gamma \left(\frac{a_0 \mu}{54} - \frac{\lambda_1}{9} \right) \right\} B_L^4
 \end{aligned}$$

15. Hub moment and torque coefficients must be resolved back to helicopter body axis in a similar manner to the force coefficients. Because torque is the dominant moment, and since A_{1W} and B_{1W} are small, the torque coefficient in wind axis is assumed to be equal to that in ANF axis so that

$$C_{Q_W} = C_Q$$

With hub moments C_{L_W} and C_{M_W} already in wind axes we now resolve successively from wind to body axes through the azimuthal angle, ψ_{WH} , and shaft angle, θ_{SH} , to give

$$C_{Q_H} = C_{Q_S} = C_{Q_W} = C_Q$$

$$C_{L_H} = C_{L_W} \cos \psi_{WH} - C_{M_W} \sin \psi_{WH} + C_{Q_S} \theta_{SH}$$

$$C_{M_H} = C_{M_W} \cos \psi_{WH} + C_{L_W} \sin \psi_{WH}$$

3.2 Tail Rotor

The main outputs required from the tail rotor model are thrust and torque. The former provides a fuselage yawing moment to oppose the main rotor torque reaction and also a side force which must be countered by a lateral tilt of the main rotor. The tail rotor torque contributes to the overall power demand on the propulsion system and its reaction also gives rise to a small fuselage pitching moment. Other tail rotor forces and hub moments are not considered significant.

The tail rotor differs from the main rotor in the following respects,

- (a) Blades are untwisted.
- (b) There is no swash plate and hence the control axis coincides with the shaft axis.
- (c) The flap hinge angle, $\delta_3 = 45$ deg, results in a blade pitch decrease of 1 deg for each degree of positive flapping.

This last feature means that cyclic flapping arising from aircraft motion will produce corresponding pitch variation with respect to the control axis which is therefore, by definition, not the ANF. Thus, the orientation of the ANF with respect to the control or shaft axis is taken into account when calculating the airflow angle of attack with

respect to the ANF system (equation 6 below, also see Ref. 9). Another effect caused by a large δ_3 is the change in effective collective pitch with coning, (equation 7), which is much greater than that for the main rotor.

While thrust and torque expressions relate to the ANF system, it is assumed that because of the small angles involved, the values of thrust and torque pertain to the hub/shaft axis system.

Calculation is essentially the same as for the main rotor but with new axes systems centred at the hub being defined. Notation changes involve the additional suffix "T" where required. Steps involved in the calculation are summarised below:

1. Airspeed components at the hub arising from aircraft translational and rotational motion in tail rotor axes. (Downwash effects from the main rotor are ignored).

$$\begin{aligned}
 u_T &= u_{HWH} - h_T q_{HEH} \\
 v_T &= w_{HWH} + l_T q_{HEH} \\
 w_T &= -v_{HWH} + l_T r_{HEH} - h_T p_{HEH}
 \end{aligned}
 \quad \text{derived in} \\
 \quad \text{KINEMATICS} \\
 \quad \text{Section 6.}$$

2. Component of incident flow in $(XOY)_T$ plane,

$$u_{TW} = \sqrt{(u_T^2 + v_T^2)}$$

3. Airspeed magnitude,

$$v_T = \sqrt{(u_{TW}^2 + w_T^2)}$$

4. Advance ratio,

$$\mu_T = \frac{v_T \cos \alpha_T}{\Omega_T R_T}$$

(13)

5. Inflow ratio,

$$\lambda_T = \mu_T \tan \alpha_T - \frac{v_T}{\Omega_T R_T}$$

6. ANF plane angle of attack with respect to incident flow,

$$\begin{aligned} \alpha_T = \tan^{-1} \left(\frac{w_T}{u_{TW}} \right) + 8\mu_T \left\{ \left(\frac{2C_{T_T}}{a_T a_T} \right) \left(\frac{\gamma_T}{32} - \frac{1}{2} \right) \right. \\ \left. + \frac{3}{16} \lambda_T \left(\frac{2}{3} - \frac{\gamma_T}{108} \right) \right\} \end{aligned}$$

7. Effective collective pitch of blade, allowing for pitch-flap coupling,

$$\theta_{TE} = \left(\theta_T - \frac{\gamma_T}{6} \lambda_T \right) / \left(1 + \frac{\gamma_T}{8} (1 + \mu_T^2) \right)$$

8. Coning angle,

$$a_{OT} = \theta_T - \theta_{TE}$$

9. Induced flow calculation (as for main rotor, ground effect omitted),

$$v_{HOVT} = \Omega_T R_T \sqrt{(C_{T_T} / 2)}$$

$$\bar{v}_T = \frac{v_T}{v_{HOVT}}$$

$$\bar{v}_T = (\bar{v}_T^2 + \bar{v}_T^2 - 2 \bar{v}_T \bar{v}_T \sin \alpha_T)^{-\frac{1}{2}}$$

(14)

$$v_T = (k_i)_T \bar{v}_T v_{HOVT}$$

where $(k_i)_T$ is the induced velocity parameter associated with the tail rotor.

10. Blade tip loss factor,

$$B_{LT} = 1 - \sqrt{(2C_{T_T}/b_T^2)}$$

Allowance for tip losses is included in the following expressions for thrust and torque.

11. Thrust coefficient,

$$C_{T_T} = \frac{a_T \sigma_T}{2} \{ \theta_{TE} \left(\frac{1}{3} + \frac{\mu_T^2}{2} (1 - \zeta_{1T}) \right) + \frac{\lambda_T}{2} \} B_{LT}^3$$

12. Torque coefficient,

$$C_{Q_T} = \frac{a_T \sigma_T}{2} \left\{ \frac{\delta}{4a_T} (1 + \mu_T^2) + \left(-\frac{1}{3} \theta_{TE} \lambda_T - \frac{\lambda_T^2}{2} - \frac{1}{4} \mu_T^2 a_{o_T}^2 \right) B_{LT}^4 \right\}$$

4. FUSELAGE AERODYNAMICS

Aerodynamic forces and moments on the fuselage and tail surfaces only become apparent at high speeds. As the tail surfaces are positioned so as to exert controlling moments, it is important to model these in order to predict trimmed attitudes which agree with flight data. This task is rendered difficult by the fact that the empennage lies in a region of complex flow interaction. The main rotor downwash is non-uniform over the fuselage, and in translational flight the wake sweepback angle varies. In addition, the tail surfaces may be immersed in the wake from the forward fuselage, including the main rotor mast and head.

Expressions used in the present model for fuselage forces and moments are mainly based on Westland's Sea King Simulator [15], while some equations are retained from the Wessex program [9]. The former expressions have been modified so as to be in dimensionless coefficient form. These relationships are based on limited wind-tunnel data for a fuselage with no rotor. To account for varying downwash influence in the tail region, empirically based functions of the wake sweepback angle are incorporated to give better agreement with flight data.

The equations for aerodynamic forces and moments acting on the fuselage and empennage, in body axes, are stated below.

1. Fuselage longitudinal force:

$$X_F = -\frac{1}{2} \rho u_{HWH} |u_{HWH}| \left[C_{DXF} S_X + (SC_D) \alpha_M^2 + (SC_D) u/c + (SC_D)_{wep} \right]$$

where $\alpha_M = \frac{w_{HWH}}{u_{HWH}} - \alpha_D$, represents the incidence deviation of the fuselage from the minimum drag value, α_D and is limited within the range ± 0.4 radian.

2. Fuselage lateral force:

$$Y_F = -\frac{1}{2} \rho V^2 C_{DYF} S_Y \sin\beta |\sin x_{LAT}|$$

$$\text{where } V^2 = u_{HWH}^2 + v_{HWH}^2 + (w_{HWH} - v)^2$$

sideslip angle, $\beta = \arctan \left[\frac{v_{HWH}}{u_{HWH}} \right]$

and $x_{LAT} = (x_{ANF} + \beta_{1W}) \sin \psi_{WH}$

with χ_{LAT} being the lateral component of the wake sweepback angle.

3. The tail fin lateral force expression is made up of two components; the first component accounts for the unstalled case whilst the second component applies to a stalled fin.

$$Y_{TV} = -\frac{1}{2} \rho (u_{HWH}^2 + v_{HWH}^2) S_{TV} a_{TV} \alpha_{TV} K_{TV}$$

$$+ \frac{1}{2} \rho w_T |w_T| S_{TV} C_{DTV} (1 - K_{TV})$$

where $\alpha_{TV} = \beta - \arctan \left[\frac{r_{HEH} \ell_{TV}}{u_{HWH}} \right]$

and $K_{TV} = 1$ for $\alpha_{TV} \leq 11^\circ$ (unstalled)

= 0 for $\alpha_{TV} > 11^\circ$ (stalled)

4. Lateral force due to tail-rotor thrust is given by:

$$Y_T = 0.925 C_{T_T} q_{ST}$$

where $q_{ST} = \rho \pi R_T^2 (\Omega_T R_T)^2$

and a thrust reduction factor of 0.925 has been applied to account for fin interference to the tail rotor (Ref. 15).

5. The fuselage normal force is given by,

$$Z_F = -\frac{1}{2} \rho \left[u_{HWH} |u_{HWH}| (SC_L)_{\alpha_Z} \alpha_Z + (w_{HWH} - v) |w_{HWH} - v| C_{DZF} S_Z \cos^2 \chi_{FA} \right]$$

where χ_{FA} , the fore-aft wake sweepback angle is given by

$$x_{FA} = (x_{ANF} + B_{1W}) \cos \psi_{WH} - \theta_{SH}$$

and α_Z , the effective fuselage angle of attack is given by

$$\alpha_Z = \frac{w_{HWH}}{u_{HWH}} - \alpha_L - f_4 \frac{v}{u_{HWH}}$$

and f_4 is an empirical constant.

6. Normal force on the horizontal tailplane is given by

$$z_{TH} = -\frac{1}{2} \rho (u_{HWH}^2 + w_{eff}^2) S_{TH} \alpha_{TH} \left[\frac{C_N}{\alpha_{TH}} \right]$$

with α_{TH} , the flow incidence at the horizontal tail-plane, given by

$$\alpha_{TH} = i + \arctan \left[\frac{w_{eff}}{u_{HWH}} \right]$$

and i is the horizontal tail-plane incidence setting.

The effective tail-plane lift curve slope, $\frac{C_N}{\alpha_{TH}}$ is given by Fig. 2 while the effective tail-plane normal velocity, w_{eff} , is obtained from

$$w_{eff} = w_{HWH} + q_{HEH} l_{TH} + w_i (\cos \text{OLDB}_{1S} - \theta_{SH}) f(\mu_x)$$

where OLDB_{1S} is the cyclic pitch control input (see 8) and the normal component of the main rotor downwash, w_i is given by

$$w_i = EK_i \Omega R (\lambda - \mu \tan \alpha_W)$$

EK_i is an empirical factor representing the magnitude of the downwash in the region of the tail plane

and $f(\mu_x)$ is given by Fig. 3, where

$$\mu_x = u_{HWH}/\Omega R$$

The function, $f(\mu_x)$, is empirically derived and represents the varying downwash influence on the tail-plane as a function of forward speed.

With the exception of the fuselage pitching moment, the fuselage and empennage moments are expressed as products of a force and its estimated moment arm from the centre of gravity. The moments are stated below.

7. Tail rotor rolling moment:

$$L_T = Y_T h_T$$

8. Fuselage rolling moment:

$$L_F = Y_F h_F$$

9. Tail fin rolling moment:

$$L_{TV} = Y_{TV} h_{TV}$$

10. Horizontal tail-plane rolling moment:

$$L_{TH} = Z_{TH} d_{TH}$$

11. Fuselage pitching moment is given by,

$$M_F = \frac{1}{2} \rho \left[-(w_{HWH} - v) |w_{HWH} - v| (SC_M)_{BOD} + u_{HWH} |u_{HWH}| ((SC_M)_{u/c} + (SC_M)_{wep}) \right]$$

12. Pitching moment due to horizontal tail-plane is given by

$$M_{TH} = Z_{TH} \ell_{TH}$$

13. Tail Rotor yawing moment:

$$N_T = - Y_T \ell_T$$

14. The combined fuselage and tail fin yawing moment is given by:

$$N_F = - Y_F X_{yfm} + N_r r_{HEH}$$

where the moment arm X_{yfm} varies with sideslip angle β , as shown in Fig. 4, and N_r , the stability derivative, ($= \partial N / \partial r_{HEH}$) as given in Fig. 5, is a function of both forward speed and the yawing sense.

Pitch and yaw damping of the fuselage and empennage are included in expressions 12 and 14 above, while other terms, such as roll damping, are assumed to be negligible.

5. EQUATIONS OF MOTION

In order to derive the helicopter motion by use of the standard equations of motion for a rigid body in helicopter body axes, all the forces and moments derived in the previous sections must be combined to give the total aerodynamic forces (X,Y,Z) and moments (L,M,N).

Thus for the forces we have the following equations -

$$X = X_F - C_{H_H} q_S + T_{XH}$$

$$Y = Y_F + Y_T + Y_{TV} + C_{Y_H} q_S + T_{YH}$$

$$Z = Z_F + Z_{TH} - C_{T_H} q_S + T_{ZH}$$

where T_{XH} , T_{YH} , T_{ZH} are respective components of the sonar cable tension (derived in cable package [8]), and $q_S = \rho \pi R^2 (\Omega R)^2$ is a dimensionalizing factor.

Similarly for the moments we have,

$$L = q_S (C_{Y_H} h_r + C_{LH} R) + L_T + L_F + L_{TH} + L_{TV}$$

$$M = q_S (C_{H_H} h_r - C_{T_H} x_{CH} + C_{M_H} R) - C_{Q_T} q_{ST} R_T + M_F + M_{TH}$$

$$N = -C_{Y_H} q_S x_{CH} + N_T + N_F + \text{Torque}$$

where h_r and x_{CH} are appropriate aircraft dimensions related to respective moment arms and

$$q_{ST} = \rho \pi R_T^2 (\Omega_T R_T)^2 \text{ for the tail rotor}$$

Torque, Q = total aerodynamic torque of the main rotor

$$= q_S R C_{Q_S} + Q_{CMP}$$

where Q_{CMP} is the compressibility torque of the main rotor (see § 7).

The standard equations of motion are,

$$\dot{u}_{HEH} = X/m + r_{HEH} v_{HEH} - q_{HEH} w_{HEH} - g \sin\theta_{HE}$$

$$\dot{v}_{HEH} = Y/m + p_{HEH} w_{HEH} - r_{HEH} u_{HEH} + g \sin\theta_{HE} \cos\theta_{HE}$$

$$\dot{w}_{HEH} = Z/m + q_{HEH} u_{HEH} - p_{HEH} v_{HEH} + g \cos\theta_{HE} \cos\theta_{HE}$$

$$\dot{p}_{HEH} = \frac{L}{A} - \left(\frac{C-B}{A} \right) q_{HEH} r_{HEH} + q_{HEH} \Omega - \frac{I_{HUB}}{A}$$

$$\dot{q}_{HEH} = \frac{M}{B} - \left(\frac{A-C}{B} \right) p_{HEH} r_{HEH} - p_{HEH} \Omega - \frac{I_{HUB}}{B}$$

$$\dot{r}_{HEH} = \frac{N}{C} - \left(\frac{B-A}{C} \right) p_{HEH} q_{HEH} + \dot{\Omega} \frac{I_T}{C}$$

where I_T = the total main rotor moment of inertia about the shaft,

and I_{HUB} = the moment of inertia of the main rotor about the shaft, inboard of the flapping hinges

The last term in the \dot{p} and \dot{q} equations above accounts for hub gyroscopic effects, while the last term in the \dot{r} equation accounts for rotational acceleration of the main rotor system (assuming a rigid lag hinge). Engine gyroscopic terms can also be included in the above equations but are generally negligible.

The linear velocities $(u, v, w)_{HEH}$ and angular rates $(p, q, r)_{HEH}$ are found by integrating the above equations.

Inertial accelerations are given by the standard equations:

$$a_{XH} = \frac{X}{m} - g \sin\theta_{HE}$$

$$a_{YH} = \frac{Y}{m} + g \sin\theta_{HE} \cos\theta_{HE}$$

$$a_{ZH} = \frac{Z}{m} + g \cos\theta_{HE} \cos\theta_{HE}$$

where $\frac{x}{m}$, $\frac{y}{m}$, $\frac{z}{m}$ are the respective accelerations* measured by accelerometers located at the aircraft c.g.

6. KINEMATICS

The Kinematics section of the program, shown schematically in Fig. 6 is virtually unchanged from the Wessex program [9] with Euler angles being used to specify the helicopter attitude with respect to earth axes. Use of this convention enables vector components to be resolved to other axis systems by successive matrix rotations in the correct order. Each of the rotations is represented by a 3×3 matrix and these are given in Appendix 2.

The following Kinematic equations express the Euler angular rates from whence the angles are found by integration,

$$\dot{\theta}_{HE} = q_{HEH} \cos\phi_{HE} - r_{HEH} \sin\phi_{HE}$$

$$\dot{\phi}_{HE} = p_{HEH} + \dot{\psi}_{HE} \sin\theta_{HE}$$

$$\dot{\psi}_{HE} = (q_{HEH} \sin\phi_{HE} + r_{HEH} \cos\phi_{HE})/\cos\theta_{HE}$$

Referring to Fig. 6 it may be seen that the helicopter airspeed components needed for the calculation of fuselage aerodynamic forces and moments (see § 3) are obtained from the difference between the the inertial velocities $(u, v, w)_{WEH}$ and the ambient wind, stated in body axes components $(u, v, w)_{WEH}$. The latter are obtained by resolving the earth-referenced ambient values using the relation below, with the rotation matrices ϕ , θ , ψ as given in App. 2.

$$\begin{vmatrix} u \\ v \\ w \end{vmatrix}_{WEH} = \begin{vmatrix} u \\ v \\ w \end{vmatrix}_{WEE}$$

* Accelerometer readings are called a_{long} , a_{lat} , a_{norm} in flight trials (Ref.12).

where

$$u_{WEE} = -v_{WE} \cos \psi_{WEE}$$

$$v_{WEE} = -v_{WE} \sin \psi_{WEE}$$

and w_{WEE} is given along with v_{WE} and ψ_{WEE} as input data. Thus, the required airspeed components, in body axes, are

$$u_{HWH} = u_{HEH} - u_{WEH}$$

$$v_{HWH} = v_{HEH} - v_{WEH}$$

$$w_{HWH} = w_{HEH} - w_{WEH}$$

The inverse matrix conversion is used to find the aircraft trajectory by resolving the inertial velocities, obtained from the equation of motion, from body axes back to earth axes, thus

$$\begin{vmatrix} u \\ v \\ w \end{vmatrix}_{HEE} = \begin{vmatrix} \psi_{HE}^{-1} & \theta_{HE}^{-1} & \phi_{HE}^{-1} \end{vmatrix} \begin{vmatrix} u \\ v \\ w \end{vmatrix}_{HEH}$$

where the inverse matrices are given in Appendix 2.

The position co-ordinates $(x, y, z)_{HEE}$ are then obtained by integration.

The main rotor aerodynamics (see § 3.1) requires airspeed components and pitch and roll rates to be specified in shaft/wind axis. As a first step the body-referenced airspeeds are resolved into shaft axes as given below,

$$u_{HWS} = u_{HWH} - w_{HWH} \theta_{SH}$$

$$v_{HWS} = v_{HWH}$$

$$w_{HWS} = w_{HWH} + u_{HWH} \theta_{SH}$$

where small angle approximations are used for the shaft tilt angle, θ_{SH} .

Wind axes are now defined by an azimuthal rotation ψ_{WH} of the shaft axes about the OZ axis, in this case the shaft axis, so that the incident wind vector is contained in the XOZ plane of the wind axes. We have

$$\begin{aligned} u_{HWW} &= \sqrt{(u_{HWS}^2 + v_{HWS}^2)} \\ v_{HWW} &= 0 \quad \text{(by definition)} \\ w_{HWW} &= w_{HWS} \end{aligned}$$

and the azimuthal angle is given by

$$\psi_{WH} = \arctan (v_{HWS}/u_{HWS})$$

The required angular rates in wind axes are similarly derived to give,

$$\begin{aligned} p_{HWW} &= (p_{HEH} - r_{HEH} \theta_{SH}) \cos \psi_{WH} + q_{HEH} \sin \psi_{WH} \\ q_{HWW} &= - (p_{HEH} - r_{HEH} \theta_{SH}) \sin \psi_{WH} + q_{HEH} \cos \psi_{WH} \end{aligned}$$

The final quantities provided by the kinematics section are the linear velocity components of the airflow relative to the tail rotor hub shaft axes system (see Appendix 1). These airspeeds are needed for calculation of the tail rotor aerodynamics. We have,

$$\begin{aligned} u_{TWT} &= u_{HWH} - h_T q_{HEH} \\ v_{TWT} &= w_{HWH} + l_T q_{HEH} \\ w_{TWT} &= -v_{HWH} + l_T r_{HEH} - h_T p_{HEH} \end{aligned}$$

7. ENGINE MODEL

The main requirement of the engine model is to simulate the variations in torque and rotor speed (r.r.p.m) observed in flight testing. It is of particular importance in dynamic response tests involving collective and pedal inputs and of lesser importance for cyclic inputs. The collective inputs produce immediate torque response and subsequent fuel flow management to maintain the set rotor revolutions per minute (r.r.p.m). The pedal inputs produce yawing rates which are sensed as r.r.p.m. changes by the aircraft-based tachometer. Control inputs to the cyclic channels produce pitch and roll rates which modify rotor loadings. However, the ensuing rotor speed variations are much less than for the collective and pedal cases.

Detailed modelling of fuel management and speed control systems may be quite complicated [17] when the stiffness, inertia and damping of all the transmission components is allowed for. The present simplified model, shown in block diagram Fig. 7, contains some features of Padfield's simplified model [18] notably the lead-lag representation of the engine response. However, some difficulties have been experienced in choosing appropriate values for the time constants which have been estimated by trial and error.

The diagram is largely self-explanatory, the following equations defining quantities either input to the model or certain outputs restated.

Total torque load on the engine, i.e. main rotor, tail rotor, accessories, etc.

$$Q_{LOAD} = (Q + G_{RAT} Q_T) FG_{LOSS}$$

where $Q = C_{Q_S} R q_S + Q_{CMP}$

and $Q_T = C_{Q_T} R_T q_{ST} + (Q_T)_{CMP}$

with $G_{RAT} = \Omega_T / \Omega$, FG_{LOSS} = factor representing losses to transmission and accessories.

The torque contribution arising from compressibility effects is given by the following empirically based expression -

Hover case ($\mu < 0.035$)

$$Q_{CMB} = 0.25 \Delta M^2 q_s R \sigma (1 - \mu/0.035)$$

where

$$\Delta M = (M_{TIP} - 0.78) + 1.25 C_T/\sigma$$

and

$$M_{TIP} = \text{Mach number at blade tip} = \Omega R/a_s$$

Forward flight case ($\mu \geq 0.035$)

$$Q_{CMB} = 1/8 (0.2 \Delta M^3 + 0.0085 \Delta M) q_s R \sigma (1 + 4.7 \mu^2)$$

where

$$\Delta M = (M_{TIP} - 0.82) + 2.4 C_T/\sigma$$

Here M_{TIP} refers to the tip Mach number of the advancing blade,
 $= (V + \Omega R)/a_s$

Similar expressions are used in the tail rotor torque calculation.

Engine output torque is redefined for comparison with flight data (i.e. aircraft instrument) by the following

$$Q_x = (Q_{ENG}/Q_{MAX}) \times 111.0$$

where the torque at maximum engine power and specified speed is given by,

$$Q_{MAX} = (SHP_{MAX} \times 550) / (RPM_{SET} \times \frac{2\pi}{60})$$

The engine response time constants τ_2 and τ_3 are specified as follows,

$$\tau_2 = 0.008 - 0.00045 Q_{\%}$$

$$\tau_3 = 0.06 - 0.005 Q_{\%}$$

Rotor speed (r.r.p.m.) as sensed by the aircraft instrument, is the sum of the rotor angular velocity and fuselage yawing rate.

The reference r.p.m. is dependent on the collective stick setting, thus providing an approximate droop cancelling effect. No derivative term is presently used to model the anticipator action.

A first order lag (time constant τ_1) represents the fuel flow response to r.p.m. errors, whilst, as stated previously, the engine response is represented by a lead-lag term.

The total fuel flow output is an approximation to values given in the Sea King Operating Data Manual (ODM) [19].

8. CONTROL INPUTS

As with most helicopters, the Sea King incorporates a measure of control phasing, mainly to reduce the degree of coupling between the aircraft pitch and roll response to pure longitudinal or lateral cyclic control inputs. This phasing is achieved by the geometric arrangement of the pitch link attachments to the swash plate and blade cuffs. For the Sea King, a value of phase angle, $\kappa = 17.5$ deg is used, which means that for a fore-aft jack movement the blade pitch changes are greatest when the blade is at an azimuthal location of $\psi = 90 + 17.5 = 107.5$ deg. In the present model, an allowance has been made for the steady state blade lag angle, ξ , ($= k_4 Q / \Omega^2$) which reduces the effective phase angle.

The systems package of the Sea King model [3] generates cyclic blade angles A_{1S} and B_{1S} which are input to the Aerodynamics package presently described. This notation has been retained from the Wessex model where no allowance for control phasing was made and A_{1S} , B_{1S} therefore truly represented the cyclic pitch variation with respect to shaft axes. In the present case, with control phasing, if we represent the control system inputs by OLD A_{1S} and OLD B_{1S} then the effective control inputs with respect to shaft axes would be,

$$A_{1S}' = \text{OLD } A_{1S} \cos(\kappa - \xi) - \text{OLD } B_{1S} \sin(\kappa - \xi)$$

$$B_{1S}' = \text{OLD } B_{1S} \cos(\kappa - \xi) + \text{OLD } A_{1S} \sin(\kappa - \xi)$$

If wind axes are displaced azimuthally by ψ_{WH} then the control inputs with respect to wind axes are,

$$A_{1W}' = A_{1S}' \cos\psi_{WH} - B_{1S}' \sin\psi_{WH}$$

$$B_{1W}' = B_{1S}' \cos\psi_{WH} + A_{1S}' \sin\psi_{WH}$$

In the program, the above relations are combined in a simplified expression by defining an effective azimuthal angle,

$$\psi_{WEFF} = \psi_{WH} + \kappa - \xi$$

$$\text{thus, } A_{1W}' = \text{OLD } A_{1S} \cos\psi_{WEFF} - \text{OLD } B_{1S} \sin\psi_{WEFF}$$

$$B_{1W}' = \text{OLD } B_{1S} \cos\psi_{WEFF} + \text{OLD } A_{1S} \sin\psi_{WEFF}$$

where the angles A_{1W}' , B_{1W}' represent the orientation of the control axis with respect to the wind axis. Because of pitch flap coupling the true

ANF is displaced from the control axis and hence the orientation of the ANF to the wind axis must be redefined. We have,

$$A_{1W} = A_{1W}' - k_3 a_1$$

$$B_{1W} = B_{1W}' - k_3 b_1$$

where k_3 is the pitch-flap coefficient associated with the δ_3 hinge.

An additional effect of blade pitch-flap and pitch-lag coupling is to modify the collective pitch input from the control system. We have,

$$\theta_c = \text{OLD } \theta_c - k_3 a_0 - k_5 \xi$$

where a_0 , ξ are the steady state blade flap and lag angles, and k_5 is the pitch-lag coefficient associated with the a_1 hinge.

9. DISCUSSION

At this stage, only limited comparisons of model prediction against flight trials data have been attempted. In the case of dynamic response tests these have been limited to autostabilizer 'OFF' conditions where the aircraft responses are dependent only on the aerodynamics, and control system corrections are absent. A full validation program is under way which will also cover autostabilizer 'ON' cases and ASW operations.

Some typical examples of the model capabilities and pre-validation limitations are given in the following Figures. In the first case, flight data [18] for steady trimmed flight are compared with model predictions. In Figs. 8a,b,c, the torque, roll attitude and pitch attitude variations with airspeed show reasonable agreement.

Examples of the engine model are given in Fig 9a, b, where the transient torque and rotor r.p.m. variations following a collective pitch pulse input are modelled quite well in Fig 9a. On the other hand Fig. 9b

shows that for a pedal step input the engine transients are much smaller than predicted by the model, although the yaw rate response shows good agreement.

Of the final examples, Fig 10a shows the variation of normal acceleration following a step change in collective pitch. In this instance much higher initial loads are present in the flight data because of the time required for the rotor inflow to readjust; so-called "inflow damping". As can be seen, the quasi-steady rotor representation does not cope with rapid transients during the first second. The use of a 'lagged' inflow in the model offers only partial improvement in predicting the initial high loading.

On the other hand, Fig. 10(b) shows results for a slower input (pulse). Model predictions are good for the initial portion of the pulse but the need for the lagged inflow is evident for the more rapid completion of the pulse. Further investigations of the effects of inflow lag are being undertaken.

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APPENDIX 1

AXES SYSTEMS

All axes systems are right handed orthogonal

1. Earth (inertial axes)

The origin is any arbitrarily chosen point fixed in the Earth.

The X_E axis is horizontal along the intended heading.

The Y_E axis is horizontal, normal to the X_E axis and positive to the right when looking along the X_E axis.

The Z_E axis is vertical and positive downwards.

If desired, the X_E , Y_E axes may be aligned North and East respectively.

2. Body axes

The origin is at the helicopter c.g. position.

The X_H axis lies in the fuselage plane of symmetry, parallel to the floor datum, positive forward.

The Y_H axis is normal to the X_H axes and positive to starboard.

The Z_H axis is mutually perpendicular positive downwards.

APPENDIX 1 (CONT.)

3. Shaft axes

As for body axes, except that the system is rotated θ_{SH} about the body Y_H axis so that OZ_S coincides with the shaft direction and is positive downwards.

4. Wind/Shaft axes

The origin is at the intersection of the rotor flapping hinge plane and the shaft.

The shaft axes are rotated about OZ_W so that the apparent wind vector is contained in the $X_W Z_W$ plane.

The Z_W axis lies along the shaft in the direction and sense of Z_S .

In this system of axes blade pitch variation is given by the relationship

$$\theta_W(\psi, r) = \theta_O(r) - A_{1W} \cos\psi - B_{1W} \sin\psi$$

5. ANF axes

If A_{1W} and B_{1W} are the components of cyclic pitch variation with respect to the wind axes, then the ANF orientation is arrived at by successive rotations of A_{1W} and $-B_{1W}$ about the OX_W and the new OY_{ANF} respectively.

6. Tail Rotor Axes Systems

(a) Hub centred shaft axes,

The X_T axis is parallel to OX_H , positive forward.

APPENDIX 1 (CONT.)

The Y_T axis is parallel to OZ_H , positive downwards.

The Z_T axis is parallel to OY_H , positive to port.

- (b) Wind/shaft axes are obtained from the shaft axes by a rotation about OZ_T such that the incident flow vector is contained in the resultant (XOZ) plane.
- (c) The ANF axes orientation is established by successive rotation (as for the main rotor) through angles representing the cyclic components observed with respect to the wind axes. For the tail rotor these pitch changes result from pitch flap coupling.

APPENDIX 2

MATRIX TRANSFORMS AND INVERSES

The following rotation matrices and their inverses are employed in the KINEMATICS section. The inverses are the same as the transposes.

$$\Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}, \quad \Phi^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \quad \Theta^{-1} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Psi^{-1} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NOTE: $\Phi^{-1} = \Phi^T$

$$\Theta^{-1} = \Theta^T$$

$$\Psi^{-1} = \Psi^T$$

$$\Psi^{-1} \Theta^{-1} \Phi^{-1} = (\Phi \Theta \Psi)^T$$

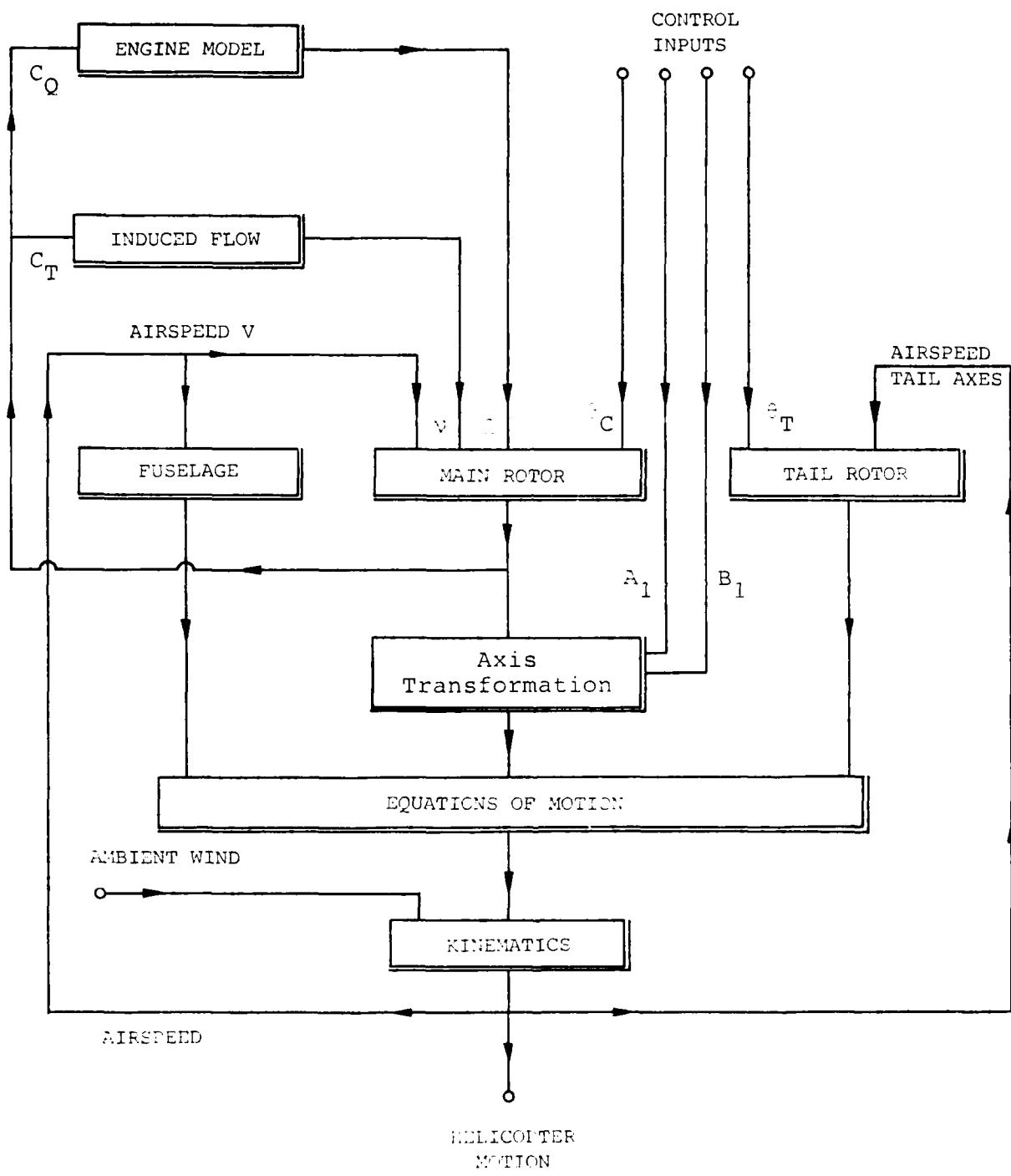


FIG. 1 STRUCTURE OF AERODYNAMICS/KINEMATICS MODEL

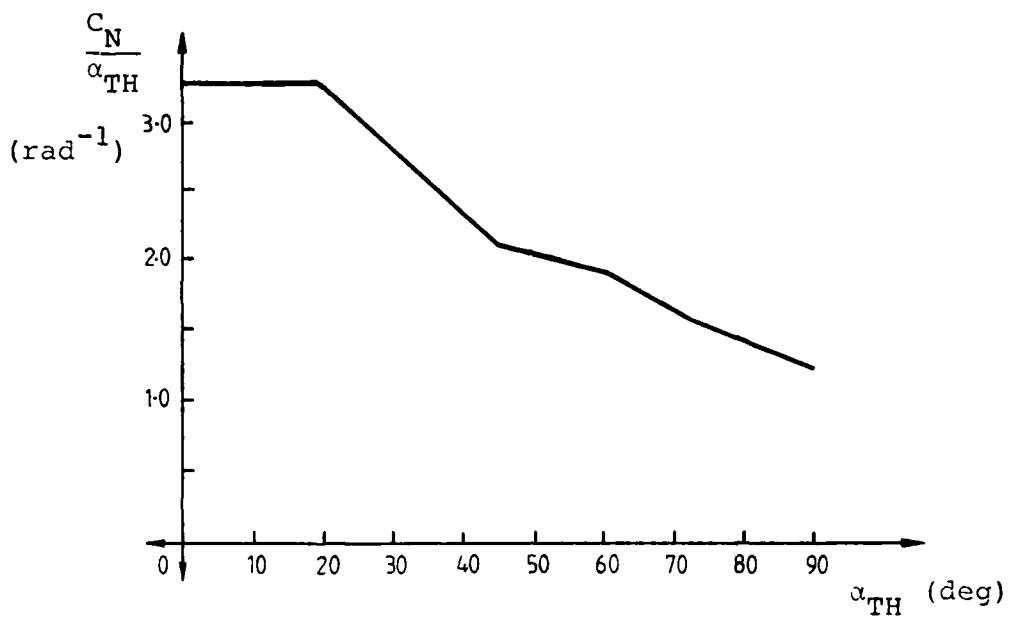


FIG. 2 TAILPLANE LIFT CURVE SLOPE VARIATION WITH ANGLE OF ATTACK

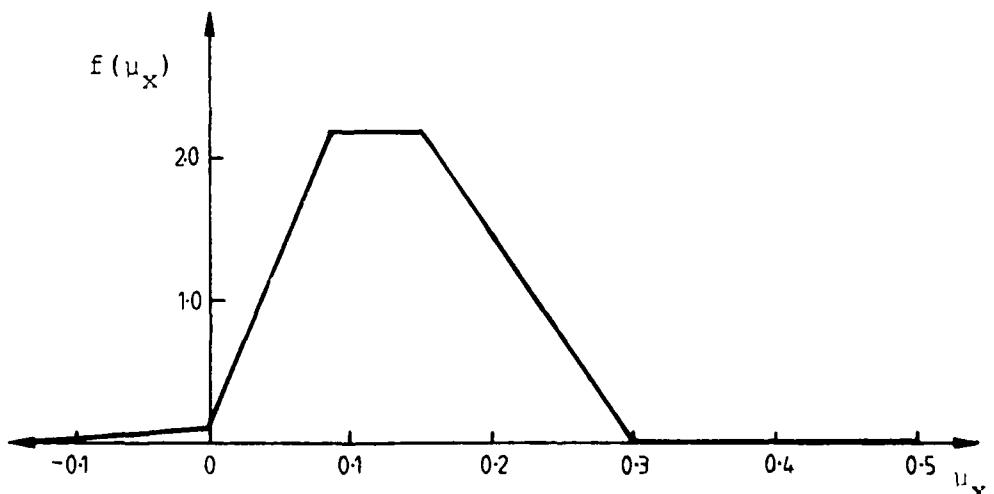


FIG. 3 TAILPLANE DOWNWASH MODIFYING FUNCTION VS FUSELAGE ADVANCE RATIO

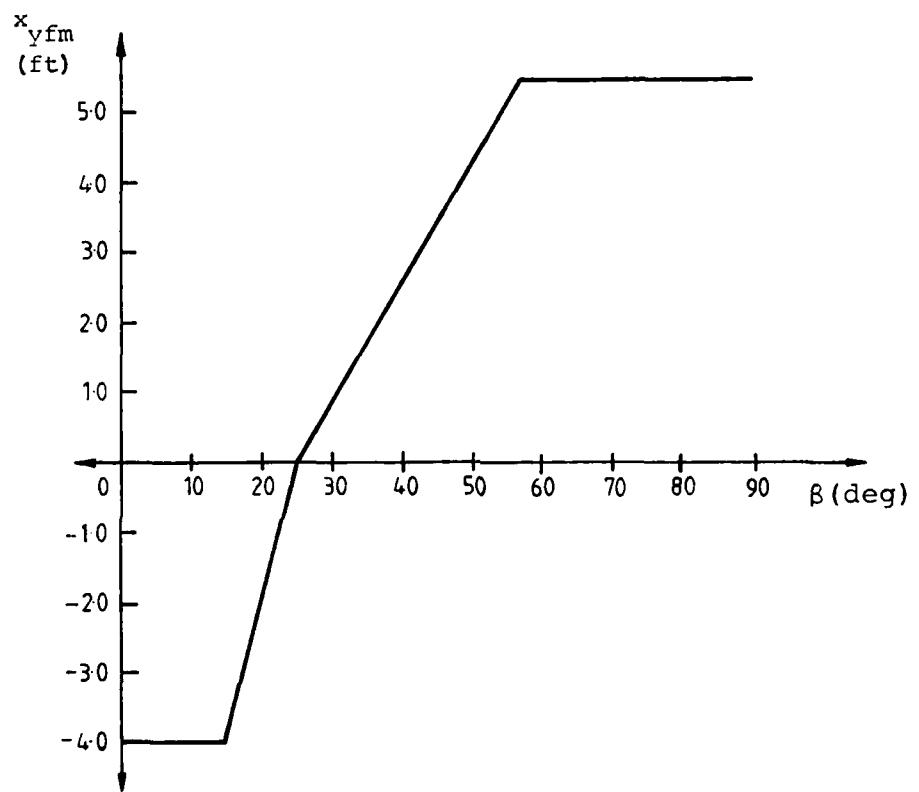


FIG. 4 LOCATION OF FUSELAGE SIDE FORCE CENTRE OF PRESSURE AS A FUNCTION OF SIDESLIP ANGLE

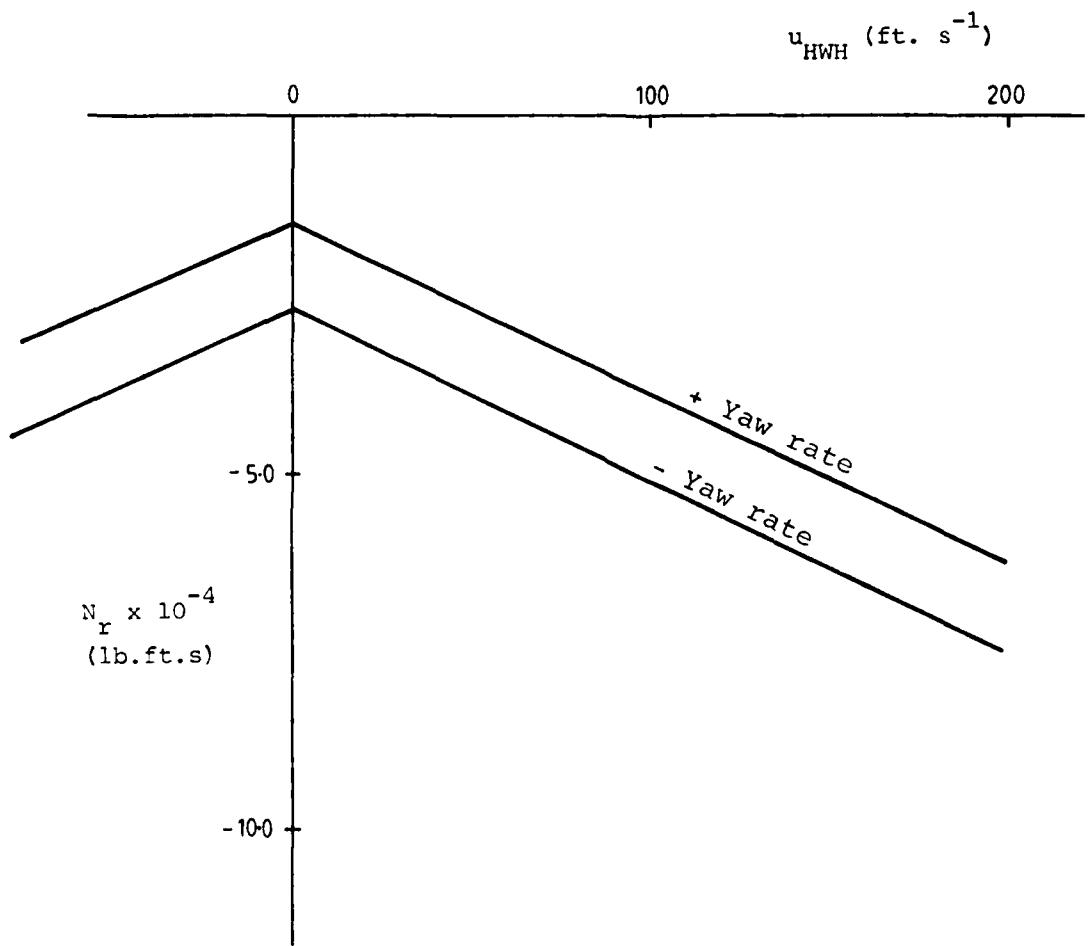


FIG. 5 VARIATION OF YAW RATE STABILITY DERIVATIVE (N_r) WITH AIRSPEED.

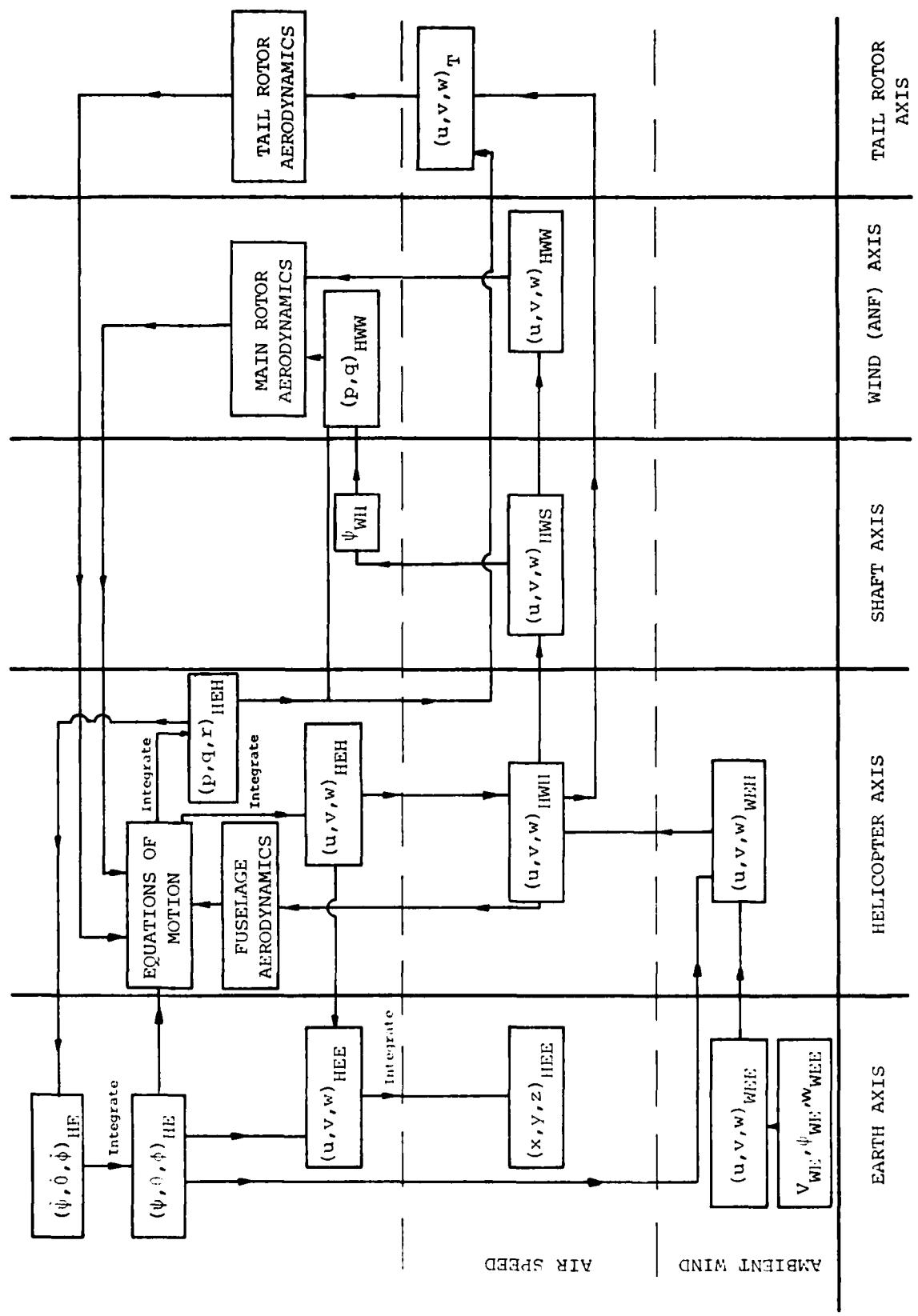
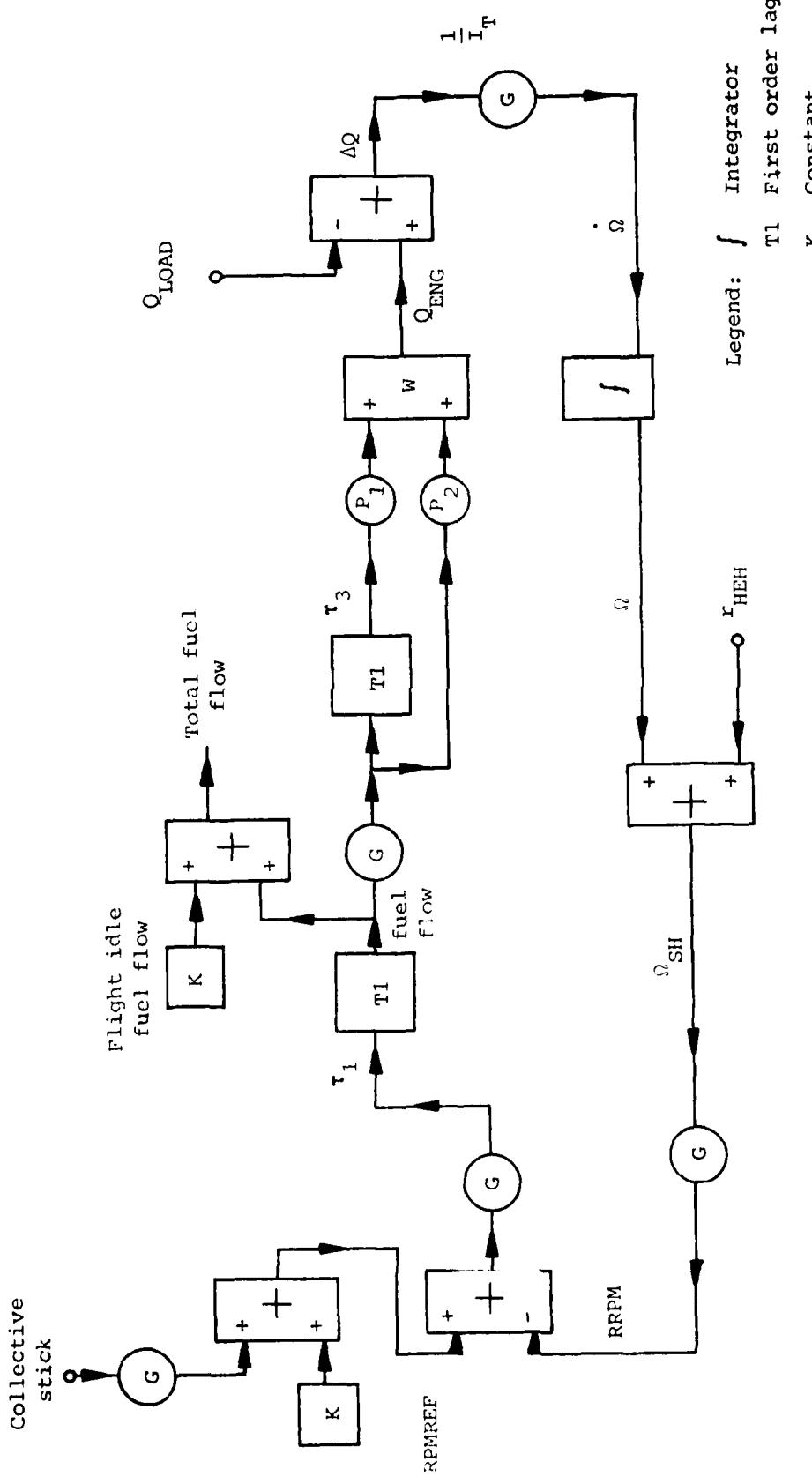


FIG. 6 INTER-RELATIONSHIP OF QUANTITIES DERIVED IN KINEMATICS (SECTION 6)



Legend:

- \int Integrator
- τ_1 First order lag
- K Constant
- G Gain
- +
 Summer
- W Weighted summer with parameters $P_1 = 1 - \tau_2 / \tau_3$
- $P_2 = \tau_2 / \tau_3$

FIG. 7 ENGINE MODEL AS SIMULATED IN CSMC-10

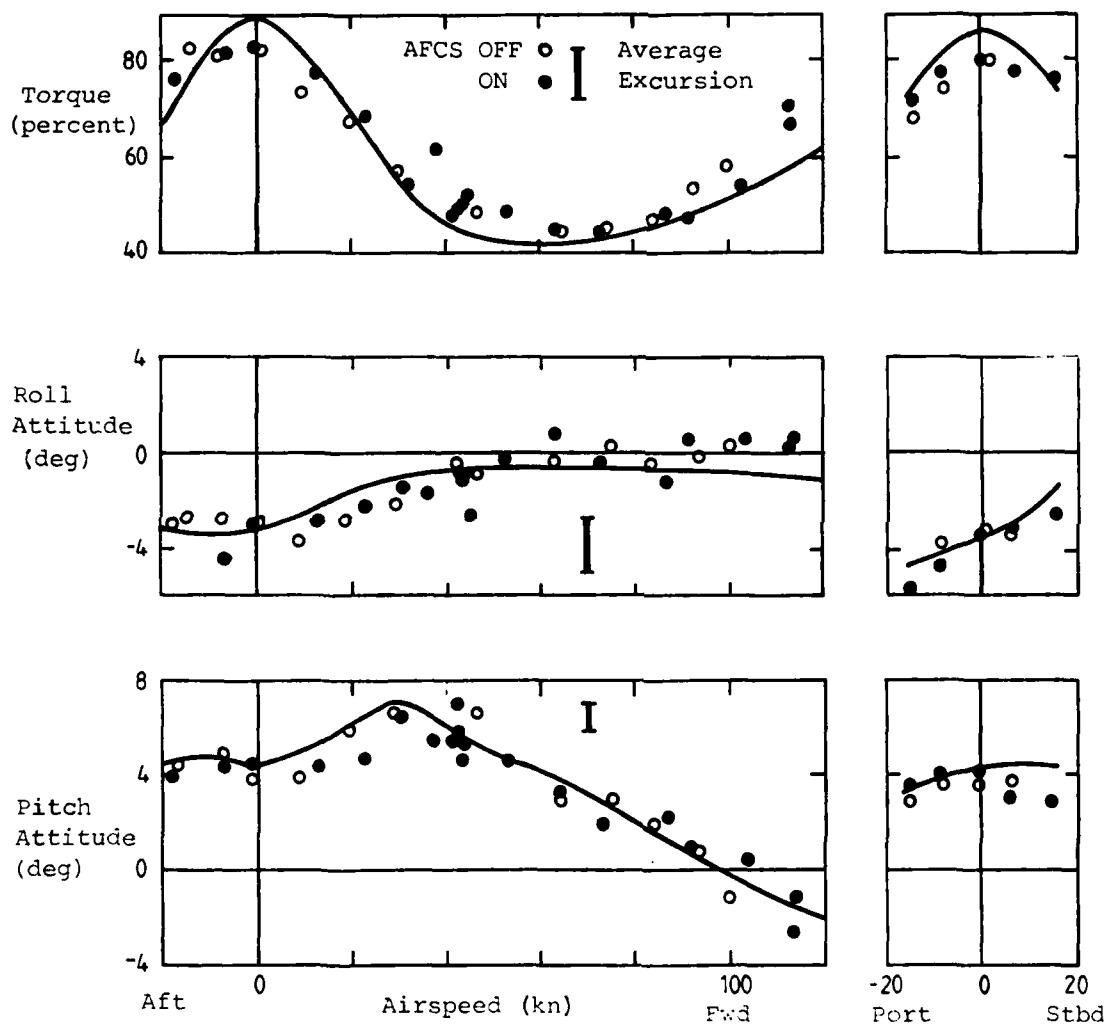
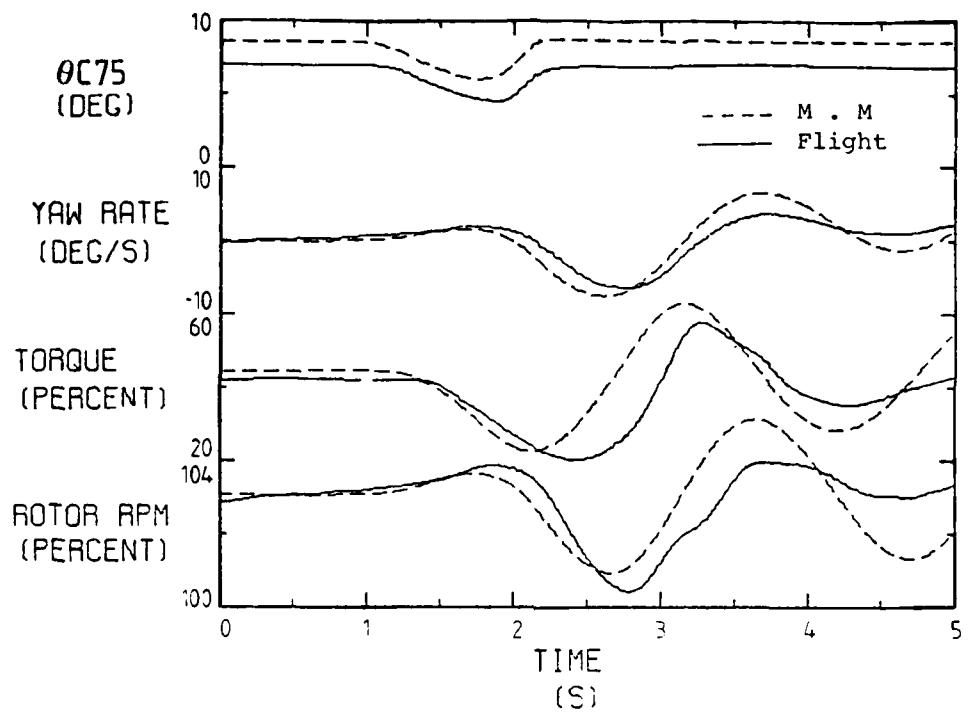
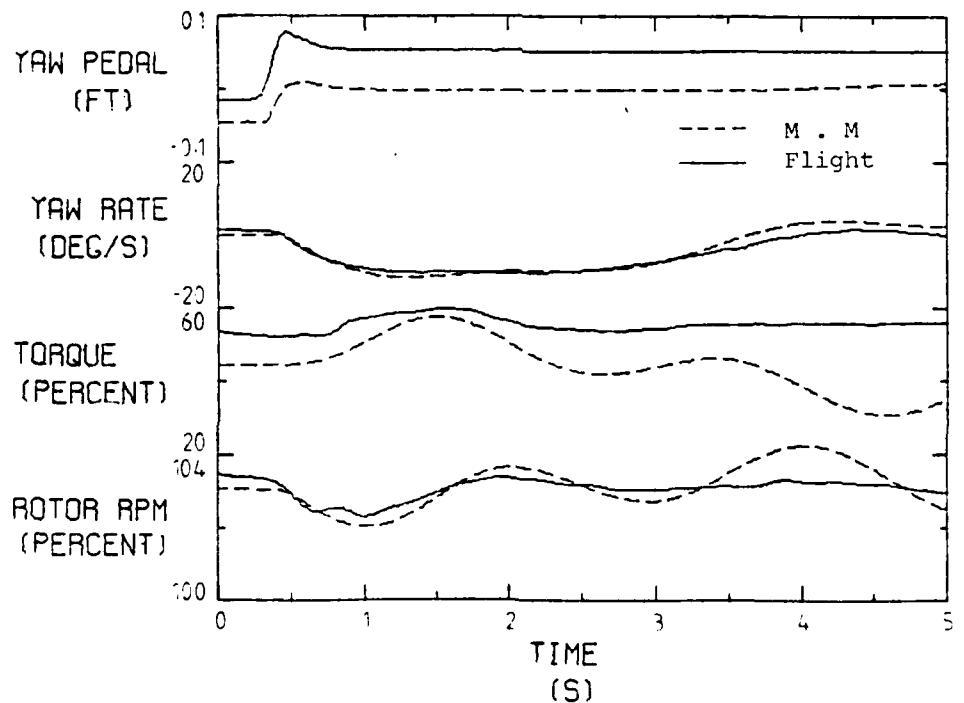


FIG. 8 COMPARISON OF MODEL PREDICTIONS WITH TRIMMED FLIGHT DATA

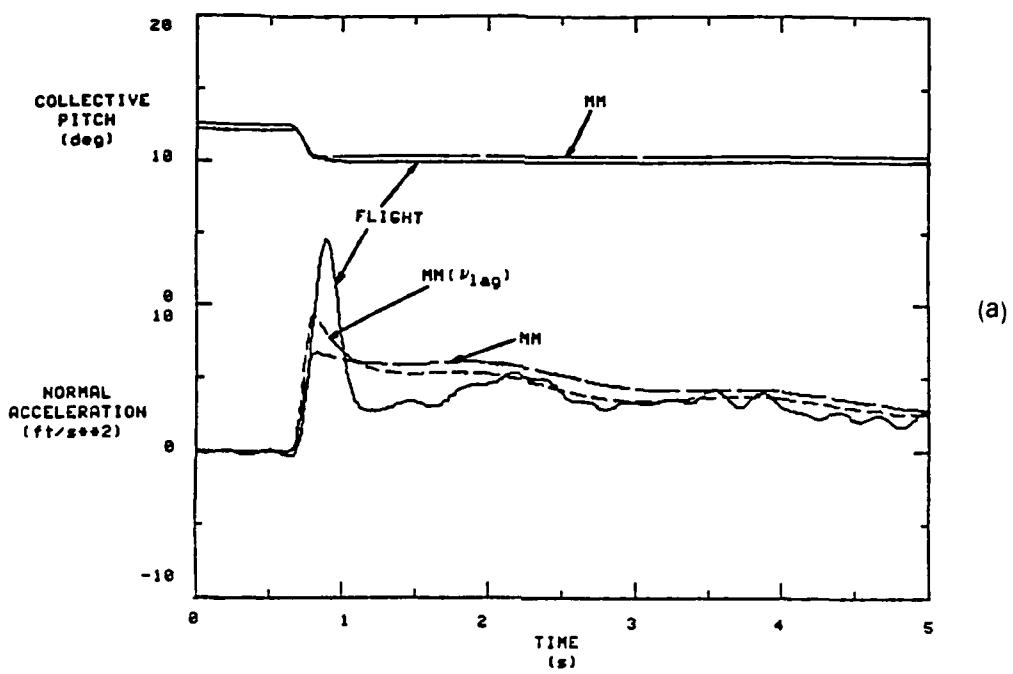


COLLECTIVE INPUT @ 40 KTS

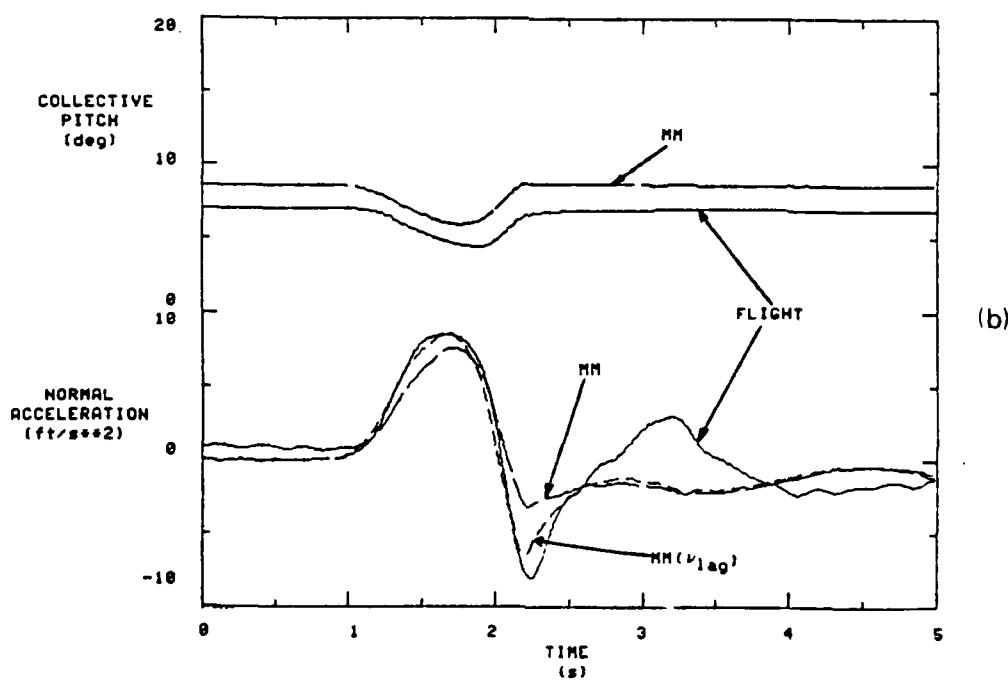


PEDAL INPUT @ 40 KTS

FIG. 9 PERFORMANCE OF ENGINE MODEL



(a)



(b)

FIG. 10 EFFECT OF INFLOW LAGGING FOR COLLECTIVE INPUTS OF DIFFERENT RATES

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